U.G. 4th Semester Examination - 2022

PHYSICS

[HONOURS]

Course Code : PHY-H-CC-T-8

Full Marks : 40 Time : $2\frac{1}{2}$ Hours

The figures in the right-hand margin indicate marks. Candidates are required to give their answers in their own words as far as practicable.

GROUP–A

1. Answer any **five** questions: 2×5=10

a) Find all the roots of the equation $e^{2z-1}=i$.

b) Find an analytic function whose real part is $u = \sin x \sinh y$.

c) Write down the Taylor series for $f(z)=\frac{1}{e^{-z}}$ at $z = 0$.

d) Find the singular points of the function $\frac{(z-2)}{(z+1)(z^2+1)}$.

e) Find the Fourier transform of 1.

f) Find the convolution of $F(t)$ with delta function $\delta(t-t_0)$.

g) Find the Laplace transform of the function $t^2 \sin at$.

h) Find the Laplace transformation of the function $\delta(t-t_0)$.

GROUP–B

2. Answer any **two** questions from the following: 5×2=10

a) Evaluate $\int \frac{e^z}{1+z^2}dz$ over the circle $|z|=2$.

b) Find the Fourier transform of

$$f(x)=\begin{cases} 1-x, & |x|<1 \\ 0, & |x|>1 \end{cases}$$

c) Write about different types of isolated singularities. Give one example each.

d) Find the inverse Laplace transform of $f(s)=\frac{6}{(s^2+9)^2}$.

GROUP–C

Answer any **two** questions from the following:

10×2=20

3\. a) Solve $x''(t)+4x'(t)+4x(t)=4e^{-2t}$ using Laplace transform when x(0) = –1 and $x'(0)=4$.

b) Find $L^{-1}\left\{ln\dfrac{s+2}{s-5}\right\}$. 5+5

4\. a) Find the Laurent Series of the function $f(z)=\dfrac{1}{(z+1)(z+3)}$ for the following regions:

i) $1<|z|<3$

ii) $|z|>3$

b) If C is a circle of radius ρ about z_0, show that $\oint_C \dfrac{dz}{(z-z_0)^n}=2\pi i$ if n = 1 but for any other integer value of n, positive or negative, the integral is zero. 5+5

5\. Using contour integration, evaluate any **two** of the following:

a) $\displaystyle\int_0^\infty \frac{dx}{x^4+1}$

b) $\displaystyle\int_0^\pi \frac{sin\,3\theta}{5-3\,cos\,\theta}\,d\theta$

c) $\displaystyle\int_0^\infty \frac{sin^2 x}{x^2}dx$ 5+5

6\. a) Solve the one-dimensional heat flow equation

$$\frac{\partial\psi}{\partial t}=\kappa^2\frac{\partial^2\psi}{\partial x^2}$$

Using Fourier transform where the solution $\psi(x,t)$ is the Temperature at position x and time t.

b) Show that $F\left\{x^m e^{-\frac{x^2}{2}}\right\}=(i)^m\dfrac{d^m}{ds^m}\left[e^{-\frac{s^2}{2}}\right]$.

5+5

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